Chapter 10: Interactions & Potential Energy April 2, 2019

↳ doing in different order than book & Simplifying

work-energy principle → $W_{ext} = \Delta E_{syst} =$

$= \Delta K + \Delta U_g + \Delta U_{sp} + \Delta E_{th}$

change in gravitational potential energy

change in elastic potential energy

April 3rd, 2019

$W_{ext} = \Delta E_{system}$

$W_{ext} = \Delta K + \Delta U_g + \Delta U_{sp} + \Delta E_{th}$ → $f_k \Delta K$

Sum of work done by all external forces (forces the environment exerts on the system

$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

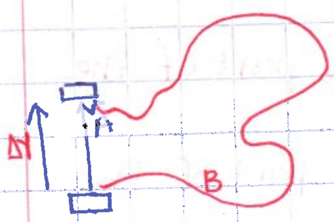
Change in gravitational potential energy

change in elastic potential energy

There are 2 kinds of forces:

Non-conservative force → Any force that isn't a conservative force

Conservative force → 1) The work done by a conservative force is path independent; the work only depends on initial & final positions.



2) You can associate a potential energy with a conservative force.

$\boxed{\Delta U = -W_{cons\, i \to f}}$ → How much work the conservative force does as object moves from initial to final position.

$U_f - U_i$

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Work done by gravity: $W_g = -mg\Delta y$

$\Delta U_g = -W_g = mg\Delta y$

$(U_g)_f - (U_g)_i = mgy_f - mgy_i$

$\boxed{U_g = mgy}$

$m \rightarrow$ mass in kg

$g \rightarrow 9.80\ m/s^2$

$y \rightarrow$ height in m

* You can define $y=0$ anywhere because only changes in potential energy are meaningful.

Work done by a spring (elastic force)

$\longrightarrow W_{sp} = -[\frac{1}{2}k(\Delta x_f)^2 - \frac{1}{2}k\,k(\Delta x_i)^2]$

$\Delta U_{sp} = -W_{sp} = 1/2\,k(\Delta x_f)^2 - \frac{1}{2}k(\Delta x_i)^2$

$\boxed{U_{sp} = 1/2\,k\Delta x^2}$

$k \rightarrow$ spring constant in Nm

$\Delta x = x - x_{eq}$

$W_{ext} = \Delta E_{sys} = \Delta K + \Delta U_g + \Delta U_{sp} + \Delta E_{th}$

$\Delta E_{th} \rightarrow f_k \Delta x$

$\Delta K \rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$\Delta U_g \rightarrow mgy_f - mgy_i$

$\Delta U_{sp} \rightarrow \frac{1}{2}k(\Delta x_f)^2 - \frac{1}{2}k(\Delta x_i)^2$

* If we include U_g, the earth must be part of the System
* If we include U_{sp}, the spring must be part of the System
* If we include ΔE_{th}, both objects that change temperature be part of the System.

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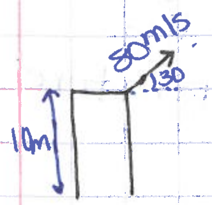
Problem 10.13



System - ball & earth

$$K_i + \underbrace{(U_g)_i}_{=0} + \underbrace{(U_{sp})_i}_{=0} + W_{ext} = K_f + \underbrace{(U_g)_f}_{=0} + \underbrace{(U_{sp})_f}_{=0} + \underbrace{\Delta E_{th}}_{=0}$$

$$K_i + (U_g)_i = K_f \rightarrow \tfrac{1}{2}mv_i^2 + mgy_i = \tfrac{1}{2}mv_f^2$$

$$\tfrac{1}{2}v_i^2 + gy_i = \tfrac{1}{2}v_f^2$$

$$v_i^2 + 2gy_i = v_f^2$$

$$v_f = \sqrt{v_i^2 + 2gy_i}$$

$$v_f = \sqrt{v_i^2 + 2gy_i}$$

$$v_f = \sqrt{80\,m/s + 2(9.80\,m/s^2)(10\,m)}$$

$$v_f = 81\,m/s$$

$$\therefore W_{ext} = \Delta E_{system} = \Delta K + \Delta U_g + \Delta U_{sp} + \Delta E_{th}$$

$$W_{ext} = (K_f - K_i) + [(U_g)_f - (U_g)_i] + [(U_{sp})_f - (U_{sp})_i] + \Delta E_{th}$$

$$\boxed{K_i + (U_g)_i + (U_{sp})_i + W_{ext} = K_f + (U_g)_f + (U_{sp})_f + \Delta E_{th}}$$

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$$\boxed{W_{ext} = \Delta E_{sys} = \Delta K + \Delta U_g + \Delta U_{sp} + \Delta E_{th}}$$

Sum of work done by all external forces

$$\boxed{W = \int_{x_i}^{x_f} F_x\,dx}$$ → applies to any force

$$\boxed{W = \vec{F}\cdot\Delta\vec{r} = |\vec{F}||\Delta\vec{r}|\cos\theta}$$ → only applies if $\vec{F}$ is constant.

$$\boxed{K = \tfrac{1}{2}mv^2}$$

$$\boxed{U_g = mgy}$$ * can define $y=0$ anywhere

$$\boxed{U_{sp} = 1/2\,k(\Delta x)^2}$$

$$\boxed{\Delta E_{th} = f_k \Delta x}$$

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→ must include earth in System if using U_g
→ must include spring in System if using U_{sp}
→ must include both objects whose temperature changes if using ΔE_{th}

$$W_{ext} = (K_f - K_i) + [(U_g)_f - (U_g)_i] + [(U_{sp})_f - (U_{sp})_i] + \Delta E_{th}$$

$$\boxed{K_i + (U_g)_i + (U_{sp})_i + W_{ext} = K_f + (U_g)_f + (U_{sp})_f + \Delta E_{th}}$$

Ex

system → puck + earth
external forces → ramp on puck (n).

$$K_i + \overset{=0 \text{ b/c } y_i=0}{(U_g)_i} + \overset{=0}{(U_{sp})_i} + \underset{\text{b/c } \theta=90° \text{ for } n}{\overset{=0}{W_{ext}}} = \overset{=0 \text{ b/c } v_f=0}{K_f} + (U_g)_f + \overset{=0}{(U_{sp})_f} + \Delta E_{th} =0 \text{ b/c no } f_k$$

$y_f = 3.0m \cdot \sin 20°$
$= 1.03m$

$K_i = (U_g)_f$

$\frac{1}{2} m v_i^2 = m g y_f$

$v_i^2 = 2 g y_f$

$v_i = \sqrt{2 g y_f}$

$v_i = \sqrt{2(9.80 m/s^2)(1.03m)}$

$v_i = 4.49 m/s$

System → puck
external forces → normal force & gravity

$$K_i + \underset{=0}{(U_g)_i} + \underset{=0}{(U_{sp})_i} + W_{ext} = \underset{=0}{K_f} + \underset{=0}{(U_g)_f} + \underset{=0}{(U_{sp})_f} + \underset{=0}{\Delta E_{th}}$$

$K_i + W_g = 0 \longrightarrow \frac{1}{2} m v_i^2 + (-mg\Delta y) = 0 \longrightarrow$

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$\frac{1}{2}mv_i^2 + (-mg\Delta y) = 0$

$\frac{1}{2}\cancel{m}v_i^2 = \cancel{m}g\Delta y \rightarrow \frac{1}{2}v_i^2 = g\Delta y \rightarrow v_i^2 = 2g\Delta y \rightarrow v_i = \sqrt{2g\Delta y}$

$v_i = \sqrt{2(9.80 m/s^2)(1.03 m)} = 4.49 m/s$

$K_i + W_g = 0$

$\frac{1}{2}mv_i^2 + (mg)(3.0m)\cos 110° = 0$

$\frac{1}{2}mv_i^2 = -mg\underbrace{(3.0m)\cos 110°}_{-1.03m}$

Note: Normal fore can do work.

Ex Problem 10.11

System → car + earth + Surface

External forces → None

$y=0$ at ground level

$K_i + (U_g)_i + \overset{=0}{W_{ext}} = K_f + (U_g)_f + \overset{=0}{\Delta E_{th}}$

$K_i + (U_g)_i = K_f + (U_g)_f \rightarrow \frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gy_i = \frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gy_f$

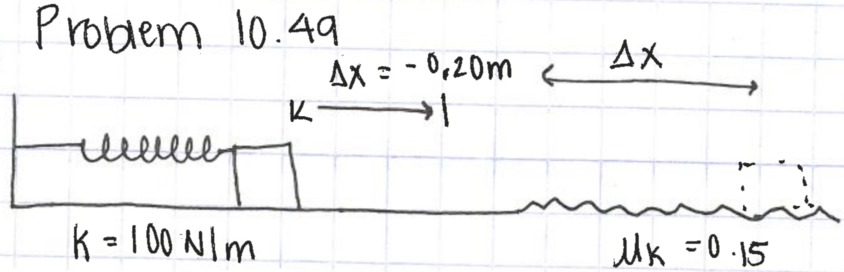
$\rightarrow 2\left(\frac{1}{2}v_i^2 + gy_i\right) = \left(\frac{1}{2}v_f^2 + gy_f\right)2 \rightarrow v_i^2 + \underset{-2gy_f}{2gy_i} = v_f^2 + \underset{-2gy_f}{\cancel{2y_f g}}$

$\rightarrow v_f^2 = v_i^2 + 2gy_i - 2gy_f \rightarrow v_f = \sqrt{v_i^2 + 2g(y_i - y_f)}$

$v_f = \sqrt{(10.0m/s)^2 + 2(9.80m/s^2(10m - 15m)}$

$v_f = 1.41 m/s$

Ex Problem 10.49



System → box + earth + Surface + Spring

External forces → None

CH 10

Problem 10.49 continued

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$U_{spi} = \Delta E_{th}$

$$\underbrace{K_i}_{=0} + \underbrace{(U_g)_i}_{=0} + (U_{sp})_i + \underbrace{W_{ext}}_{=0} = \underbrace{K_f}_{=0} + \underbrace{(U_g)_f}_{=0} + \underbrace{(U_{sp})_f}_{=0} + \Delta E_{th}$$

$(U_{sp})_i = \Delta E_{th}$

CH 10 & Recap

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problem 10.49 continued

$U_{sp_i} = \Delta E_{th}$

$$\overset{=0}{K_i} + \overset{=0}{(U_g)_i} + (U_{sp})_i + \overset{=0}{W_{ext}} = \overset{=0}{K_f} + \overset{=0}{(U_g)_f} + \overset{=0}{(U_{sp})_f} + \Delta E_{th}$$

$$(U_{sp})_i = \Delta E_{th}$$

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Recap

$$\boxed{W_{ext} = \Delta K + \Delta U_g + \Delta U_{sp} + \Delta E_{th}}$$

$$\boxed{K_i + (U_g)_i + (U_{sp})_i + W_{ext} = K_f + (U_g)_f + (U_{sp})_f + \Delta E_{th}}$$

* $$\boxed{\tfrac{1}{2} m v_i^2 + m g y_i + \tfrac{1}{2} k (\Delta x_i)^2 + W_{ext} = \tfrac{1}{2} m v_f^2 + m g y_f + \tfrac{1}{2} k (\Delta x_f)^2 + f_k \Delta x}$$

* you can define y=0 anywhere
* U_g → earth must be part of System
* U_{sp} → Spring must be part of the System
* E_{th} → both objects whose temperature increases must be part of the System

Ex

A ball dropped from rest at height y_i:
What is speed right before it hits the ground?
System → just ball

$$W_{ext} = \Delta K + \overset{=0}{\Delta U_{sp}} + \overset{=0}{\Delta E_{th}}$$

$$W_{ext} = \Delta K = (K_f - \overset{=0}{K_i})$$

$$W_g = \tfrac{1}{2} m v_f^2 \rightarrow -mg\Delta y = \tfrac{1}{2} m v_f^2$$

$$v_f = \sqrt{-2g\Delta y} = \sqrt{2 g y_i}$$

$$\Delta y = y_f - y_i = -y_i$$

Note: No ΔU_g because earth is not part of the System

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System → ball and earth

$W_{ext} = \Delta K + \Delta U_g + \cancel{\Delta U_{sp}}^{0} + \cancel{\Delta E_{th}}^{0}$

$0 = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2}^{0} + \cancel{mgy_f}^{0} - mgy_i$

$0 = \frac{1}{2}mv_f^2 - mgy_i$

$v_f = \sqrt{2gy_i}$

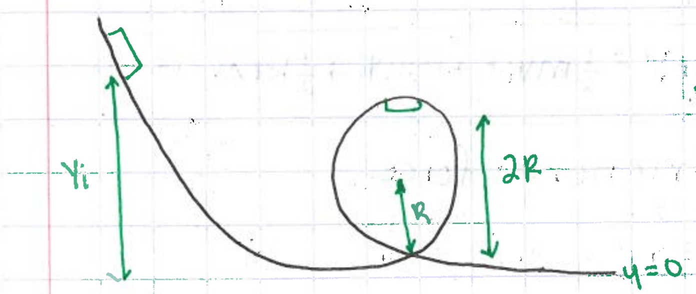
PROBLEM 10.49 CONTINUED

$(U_{sp})_i = \Delta E_{th}$

$\frac{1}{2}k(\Delta x_i)^2 = f_k \Delta r \qquad f_k = \mu_k n = \mu_k mg$

$\frac{1}{2}k(\Delta x_i)^2 = \mu_k mg \underset{?}{\underline{\underline{\Delta r}}}$

$\Delta r = \dfrac{\frac{1}{2}k(\Delta x_i)^2}{\mu_k mg} = \dfrac{\frac{1}{2}(100\frac{N}{m})(-0.20m)^2}{(0.15)(2.50kg)(9.80m/s^2)} = \underline{0.54m}$



$V_c = \sqrt{gR}$

$\downarrow a = \frac{v^2}{r}$

$\sum F_y = ma_y = m\frac{v^2}{r}$

$n + mg = m\frac{v^2}{r}$

minimum speed → $n > 0$

$n = m\frac{v^2}{r} - mg > 0$

$m\frac{v^2}{r} > mg$

$v > \sqrt{rg}$

$V_{minimum} = \sqrt{rg}$

System → block + earth

$K_i + (U_g)_i + (U_{sp})_i + W_{ext} = K_f + (U_g)_f + \underset{=0}{(U_{sp})_f} + \underset{=0}{\Delta E_{th}}$

$K_i = 0$, $(U_{sp})_i = 0$ ↳ $W_n = 0$ b/c $\theta = 90°$

$(U_g)_i = K_f + (U_g)_f$

$mgy_i = \frac{1}{2}mv_f^2 + mgy_f$

$y_i = \frac{v_f^2}{2g} + y_f \rightarrow y_f = \frac{(Rg)}{2g} + 2R$

$\frac{R}{2} + 2R = \boxed{\frac{5}{2}R}$

$y_f = 2R$

$V_f = \sqrt{Rg}$

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Mechanical Energy → Sum of kinetic + potential energies

$E_{mech} = K + U$
$= K + U_g + U_{sp}$

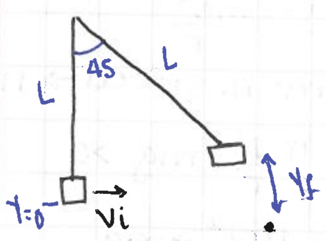
$E_{system} = E_{mech} + E_{th}$
$W_{ext} = \Delta E_{sys} = \Delta E_{mech} + \Delta E_{th}$

* if a system is isolated ($W_{ext} = 0$) → then $\Delta E_{sys} = 0$
(Conservation of energy)
* if a system is isolated ($W_{ext} = 0$) and there are no frictional forces ($\Delta E_{th} = 0$) → then $\Delta E_{sys} = \Delta E_{mech} = 0$
(Conservation of mechanical energy)

$$\boxed{\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}k(\Delta x_f)^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}k(\Delta x_i)^2}$$

↳ $W_{ext} = 0$ and no frictional force.

Problem 10.10

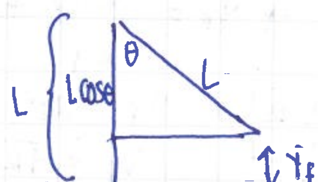


System → child + earth
external forces → tension
$W_{tension} = 0\,J$ because $\theta = 90°$

$$K_i + \cancel{(U_g)_i}^{\,0} + \cancel{W_{ext}}^{\,0} = \cancel{K_f}^{\,0} + (U_g)_f + \cancel{\Delta E_{th}}^{\,0}$$

$K_i = (U_g)_f$

$\frac{1}{2}mv_i^2 = mgy_f$



$y_f = L - L\cos\theta$
$= L(1-\cos\theta)$

$\frac{1}{2}mv_i^2 = mg(L(1-\cos\theta))$

$v_i = \sqrt{2gL(1-\cos\theta)} = \sqrt{2(9.80\,\frac{m}{s^2})(3.0\,m(1-\cos 45°))}$

$\boxed{v_i = 4.2\ m/s}$